

UnitStep

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Notations

Traditional name

Heaviside step function

Traditional notation

 $\theta(x)$

Mathematica StandardForm notation

UnitStep[x]

Primary definition

14.01.02.0001.01

$$\theta(x) = 1 \text{ ; } x \in \mathbb{R} \wedge x \geq 0$$

14.01.02.0002.01

$$\theta(x) = 0 \text{ ; } x \in \mathbb{R} \wedge x < 0$$

14.01.02.0003.01

$$\theta(x) = \begin{cases} 1 & x \geq 0 \text{ ; } x \in \mathbb{R} \end{cases}$$

Specific values

Values at fixed points

14.01.03.0001.01

$$\theta(0) = 1$$

14.01.03.0002.01

$$\theta(1) = 1$$

14.01.03.0003.01

$$\theta(-1) = 0$$

Values at infinities

14.01.03.0004.01

$$\theta(\infty) = 1$$

14.01.03.0005.01

$$\theta(-\infty) = 0$$

General characteristics

Domain and analyticity

$\theta(x)$ is a nonanalytical function; it is a piecewise constant function defined for all real x .

14.01.04.0001.01

$$x \longrightarrow \theta(x) :: \mathbb{R} \longrightarrow \mathbb{Z}$$

Symmetries and periodicities

Parity

14.01.04.0002.01

$$\theta(-x) = 1 - \theta(x) \text{ ; } x \neq 0$$

Periodicity

No periodicity

Sets of discontinuity

The function $\theta(x)$ is continuous function in $\mathbb{R} \setminus \{0\}$.

14.01.04.0003.01

$$\mathcal{DS}_x(\theta(x)) = \{0\}$$

14.01.04.0004.01

$$\lim_{\epsilon \rightarrow +0} \theta(\epsilon) = 1$$

14.01.04.0005.01

$$\lim_{\epsilon \rightarrow +0} \theta(-\epsilon) = 0$$

Series representations

Exponential Fourier series

14.01.06.0001.01

$$\theta(x) = \frac{2}{\pi} \sum_{k=0}^{\infty} \frac{\sin((2k+1)x)}{2k+1} + \frac{1}{2} \text{ ; } -\pi < x < \pi$$

Residue representations

14.01.06.0002.02

$$\theta(x) = \operatorname{res}_s \left((1-x)^{-s} \frac{1}{s} \right) (0) \text{ ; } 0 < x < 2$$

14.01.06.0003.01

$$\theta(x) = 0 \text{ ; } x < 0$$

14.01.06.0004.02

$$\theta(x) = \operatorname{res}_s \left((1+x)^{-s} \frac{1}{s} \right) (0) /; x > 0$$

14.01.06.0005.01

$$\theta(x) = 0 /; -2 < x < 0$$

Integral representations

On the real axis

Of the direct function

14.01.07.0001.01

$$\theta(x) = \frac{1}{2\pi i} \lim_{\varepsilon \rightarrow +0} \int_{-\infty}^{\infty} \frac{e^{itx}}{t - i\varepsilon} dt$$

14.01.07.0002.01

$$\theta(x) = \frac{1}{2\pi i} \lim_{\varepsilon \rightarrow +0} \int_{-\infty}^{\infty} \frac{e^{-itx}}{t + i\varepsilon} dt$$

14.01.07.0003.01

$$\theta(x) = \frac{1}{\sqrt{\pi}} \lim_{\varepsilon \rightarrow +0} \int_{-x}^{\infty} \frac{1}{t} e^{-\frac{t^2}{\varepsilon^2}} dt$$

14.01.07.0004.01

$$\theta(x) = \frac{1}{\pi} \lim_{\varepsilon \rightarrow +0} \int_{-\infty}^x \frac{1}{t} \sin\left(\frac{t}{\varepsilon}\right) dt$$

Contour integral representations

14.01.07.0005.01

$$\theta(x) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma(s) (1-x)^{-s}}{\Gamma(s+1)} ds /; 0 < \gamma \wedge x < 2$$

14.01.07.0006.01

$$\theta(x) = \frac{1}{2\pi i} \int_{\mathcal{L}} \frac{\Gamma(s) (1-x)^{-s}}{\Gamma(s+1)} ds /; x < 2$$

14.01.07.0007.01

$$\theta(x) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma(-s) (1+x)^{-s}}{\Gamma(1-s)} ds /; \gamma < 0 \wedge x > -2$$

14.01.07.0008.01

$$\theta(x) = \frac{1}{2\pi i} \int_{\mathcal{L}} \frac{\Gamma(-s) (1+x)^{-s}}{\Gamma(1-s)} ds /; x > -2$$

Limit representations

14.01.09.0001.01

$$\theta(x) = \lim_{\varepsilon \rightarrow +0} \frac{1}{e^{-\frac{x}{\varepsilon}} + 1}$$

14.01.09.0002.01

$$\theta(x) = \lim_{\varepsilon \rightarrow +0} e^{-e^{-\frac{x}{\varepsilon}}} /; x \neq 0$$

14.01.09.0003.01

$$\theta(x) = \frac{1}{2} \left(\lim_{\varepsilon \rightarrow +0} \tanh\left(\frac{x}{\varepsilon}\right) + 1 \right)$$

14.01.09.0004.01

$$\theta(x) = \frac{1}{2\pi i} \lim_{\varepsilon \rightarrow +0} (\log(i\varepsilon - x) - \log(-x - i\varepsilon))$$

14.01.09.0005.01

$$\theta(x) = \frac{1}{\pi} \lim_{\varepsilon \rightarrow +0} \tan^{-1}\left(\frac{x}{\varepsilon}\right) + \frac{1}{2}$$

14.01.09.0006.01

$$\theta(x) = \frac{1}{2} \left(\lim_{\varepsilon \rightarrow +0} \operatorname{erf}\left(\frac{x}{\varepsilon}\right) + 1 \right)$$

14.01.09.0007.01

$$\theta(x) = \frac{1}{2} \left(\lim_{\varepsilon \rightarrow +0} \operatorname{erfc}\left(-\frac{x}{\varepsilon}\right) \right)$$

14.01.09.0008.01

$$\theta(x) = \frac{1}{\pi} \lim_{\varepsilon \rightarrow +0} \operatorname{Si}\left(\frac{\pi x}{\varepsilon}\right) + \frac{1}{2}$$

14.01.09.0009.01

$$e^{i\lambda\pi(\theta(x)-\theta(-x))} = \lim_{\varepsilon \rightarrow +0} \left(\frac{\varepsilon + ix}{\varepsilon - ix} \right)^\lambda$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

14.01.16.0001.01

$$\theta(-x) = 1 - \theta(x) /; x \neq 0$$

Multiple arguments

14.01.16.0002.01

$$\theta\left(\prod_{k=1}^n (x - a_k)\right) = \sum_{k=1}^n (-1)^{n-k} \theta(x - a_k) + n - 2 \left\lfloor \frac{n-1}{2} \right\rfloor - 1 /; x \neq a_k \wedge a_k < a_{k+1} \wedge a_k \in \mathbb{R} \wedge 1 \leq k \leq n$$

Products, sums, and powers of the direct function

Powers of the direct function

14.01.16.0003.01

$$\theta(x)^a = \theta(x) /; a > 0$$

Identities

Functional identities

14.01.17.0001.01

$$\theta(b + a x) = \theta\left(-\frac{b}{a} - x\right) \theta(-a) + \theta\left(\frac{b}{a} + x\right) \theta(a) \quad ; a \in \mathbb{R} \wedge b \in \mathbb{R}$$

14.01.17.0002.01

$$\theta(x) \theta(y) = \theta(xy) \theta\left(\frac{x+y}{2}\right) \quad ; x \in \mathbb{R} \wedge y \in \mathbb{R}$$

Complex characteristics

Real part

14.01.19.0001.01

$$\operatorname{Re}(\theta(x)) = \theta(x)$$

Imaginary part

14.01.19.0002.01

$$\operatorname{Im}(\theta(x)) = 0$$

Absolute value

14.01.19.0003.01

$$|\theta(x)| = \theta(x)$$

Argument

14.01.19.0004.01

$$\arg(\theta(x)) = \tan^{-1}(\theta(x), 0)$$

Conjugate value

14.01.19.0005.01

$$\overline{\theta(x)} = \theta(x)$$

Differentiation

Low-order differentiation

14.01.20.0001.01

$$\frac{\partial \theta(x)}{\partial x} = \delta(x) \quad ; x \neq 0$$

14.01.20.0002.01

$$\frac{\partial \theta(x)}{\partial x} = \frac{\theta(x) + \theta(-x) - \delta(x) - 1}{1 - \delta(x)} \quad ; x \neq 0$$

14.01.20.0003.01

$$\frac{\partial \theta(x)}{\partial x} = \begin{cases} i & x = 0 \end{cases}$$

Integration

Indefinite integration

Involving only one direct function

14.01.21.0001.01

$$\int \theta(x) dx = x \theta(x)$$

Involving one direct function and elementary functions

Involving power function

14.01.21.0002.01

$$\int x^{\alpha-1} \theta(x) dx = \frac{x^{\alpha} \theta(x)}{\alpha}$$

14.01.21.0003.01

$$\int \frac{\theta(x)}{x} dx = \log(x) \theta(x)$$

Definite integration

14.01.21.0004.01

$$\int_{-\infty}^{\infty} \theta(t) \theta(x-t) dt = x \theta(x)$$

Integral transforms

Fourier exp transforms

14.01.22.0001.01

$$\mathcal{F}_t[\theta(t)](x) = \frac{i}{\sqrt{2\pi} x} + \sqrt{\frac{\pi}{2}} \delta(x)$$

14.01.22.0006.01

$$\mathcal{F}_t[e^{-t} \theta(t)](x) = \frac{i}{\sqrt{2\pi} (i+x)}$$

Inverse Fourier exp transforms

14.01.22.0002.01

$$\mathcal{F}_t^{-1}[\theta(t)](x) = -\frac{i}{\sqrt{2\pi} x} + \sqrt{\frac{\pi}{2}} \delta(x)$$

Fourier cos transforms

14.01.22.0003.01

$$\mathcal{F}_{C_t}[\theta(t)](x) = \sqrt{\frac{\pi}{2}} \delta(x)$$

Fourier sin transforms

14.01.22.0004.01

$$\mathcal{F}_{S_t}[\theta(t)](z) = \sqrt{\frac{2}{\pi}} \frac{1}{z}$$

Laplace transforms

14.01.22.0005.01

$$\mathcal{L}_t[\theta(t)](z) = \frac{1}{z}$$

Representations through more general functions

Through Meijer G

Classical cases for the direct function itself

14.01.26.0001.01

$$\theta(x) = G_{1,1}^{1,0} \left(1-x \left| \begin{matrix} 1 \\ 0 \end{matrix} \right. \right) ; x \in \mathbb{R} \wedge x < 2$$

14.01.26.0002.01

$$\theta(x) = G_{1,1}^{0,1} \left(x+1 \left| \begin{matrix} 1 \\ 0 \end{matrix} \right. \right) ; x \in \mathbb{R} \wedge x > -2$$

14.01.26.0003.01

$$\theta(1-|z|) = G_{1,1}^{1,0} \left(z \left| \begin{matrix} 1 \\ 0 \end{matrix} \right. \right)$$

14.01.26.0004.01

$$\theta(|z|-1) = G_{1,1}^{0,1} \left(z \left| \begin{matrix} 1 \\ 0 \end{matrix} \right. \right)$$

Representations through equivalent functions

14.01.27.0001.01

$$\theta(x) = \theta(x_1, x_2, \dots, x_n) ; x_1 = x \wedge n = 1$$

14.01.27.0002.01

$$\theta(x) = \frac{1}{2} (\operatorname{sgn}(x) + 1) ; x \in \mathbb{R} \wedge x \neq 0$$

14.01.27.0003.01

$$\theta(x) = \theta(x_1, x_2, \dots, x_n) ; x_1 = x \neq 0 \wedge n = 1$$

14.01.27.0004.01

$$\theta(x) = \theta(x) ; x \neq 0$$

History

–O. Heaviside (1881)

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